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# A new perspective in supply chain coordination

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## Abstract

Discount models have been used extensively in the past to achieve coordination between a buyer and a vendor in the context of supply chain management. Such models are based on the vendor offering a discount to the buyer so as to entice him to order in higher batch sizes. The solution is achieved at the point where the vendor is better off and the buyer is not worse off. In this paper we suggest the use of 'reverse discount' as another mechanism for coordination between the buyer and the vendor. The proposed model analyzes the coordination achieved by allowing the buyer to offer a higher price to the vendor, to motivate him in order to reduce the batch size. Various scenarios have been analyzed including determining the net savings a buyer can get through such an increase. The model has been extended to incorporate the case of information asymmetry.



## Keywords

Inventory; Supply chain management; Stochastic programming; Production; Nonlinear programming


## 1. Introduction

A supply chain is comprised of legally independent but interconnected firms, each trying to realize its own objective (Stadler, 2005). As the objectives of the firms are often conflicting in nature, independent planning by the firms may lead to supply chain inefficiencies (Cachon, 2003). In this context, the importance of supply chain coordination among the partners has been stressed by many authors in the recent past (Cachon, 2003 and Dudek, 2004). Central planning, quantity discounts, supply chain contracts (like revenue sharing, buy-back) and credit options are some of the most common mechanisms of achieving coordination in a supply chain (Cachon, 2003, Sarmah et al., 2006). These approaches of achieving coordination have been broadly classified (Albrecht, 2010) as 'strong' and 'weak' form of coordination respectively. Strong form of coordination as is discussed in Cachon (2003), is the set of actions that brings in overall supply chain optimality. Examples include Joint Economic Lot Size models (Goyal, 1977, 1988 and Banerjee, 1986), revenue sharing contracts (Cachon and Lariviere, 2005), and buy back contracts (Pasternack, 1985). Weak form of coordination (Corbett and de Groote, 2000), on the other hand, does not strive for optimality. It is a set of actions that attempts to achieve overall supply chain improvement compared to the solution that would result without these actions. Mechanisms of coordination in weak form include quantity discount models, credit options etc. Strong form of coordination is difficult to achieve in the presence of information asymmetry. This is perhaps the reason that one finds relatively more studies and applications of weak form of coordination (Albrecht, 2010). In this paper an attempt has been made to develop a new mechanism of achieving weak form of coordination.

One of the earliest attempts in achieving supply chain coordination is through quantity discounts. The traditional discounting models (Buffa and Miller, 1979) focus on buyer's response for minimizing his costs subject to the discount schedules offered by the supplier. Other discounting models such as Monahan (1984), Banerjee (1986) and Lee and Rosenblatt (1986) present the vendor's perspective to determine the vendor's quantity discount pricing schedule that will maximize his resulting economic gains without adding any further costs to the buyer. In both the approaches the buyer decides on his economic order quantity and the supplier offers a discount setting up the coordination process. Contrary to these approaches, we develop an approach where the supplier decides on lot sizes and the buyer initiates the process by offering an increase in the wholesale price to motivate the seller to deliver in smaller batch sizes. The vendors deciding on the lot sizes is quite common in many large industries like aerospace industry, automotive industry etc. where the cost of setting up for production is very high (Esmaeilli et al., 2009). As such the coordination mechanism developed may be useful whenever there is a big mismatch between the independent economic lot sizes of the vendor and the buyer. This process may be regarded as a "reverse discount" procedure. We examine the feasibility of coordination that will maximize the buyer's resulting economic gains without altering the costs of the vendor. The model has been extended to incorporate the case of information asymmetry.

The remainder of the paper is organized as follows: A brief review of the relevant literature is presented in the next section. Mathematical modeling of the problem is presented in section 3. Section 4 covers the case of information asymmetry. Numerical results are presented in section 5 and finally the conclusion is covered in section 6.

## 2. Literature Review

In a typical buyer-vendor scenario, the buyer would like to operate at his Economic Order Quantity (EOQ) based on his own inventory carrying cost and ordering cost trade-off. The vendor on the other hand, will find this order quantity to be very low (Monahan, 1984). The development of the Joint Economic Lot Size (JELS) models by Goyal (1977, 1988) and Banerjee (1986) is an attempt in supply chain coordination that aims at achieving overall supply chain optimality. These models indicate that overall gains may accrue by considering the 'supply chain point of view' rather than taking the 'buyer's perspective' alone. Detailed reviews of such models can be found in Goyal and Gupta (1989) and Sarmah et al. (2006). However, the JELS solution is not always in the best interests for both the partners as the overall system's improvement is often accompanied by differential benefits to the different partners of the supply chain (Lu, 1995; Sucky, 2005, 2006; Darwish & Odah, 2010). Thus, one of the partners will always be reluctant to go for the JELS models unless being assured of some form of compensation (Goyal and Gupta, 1989). The discounting models assume importance in this context as these models can assure the compensation for the other partner.

The traditional discount models focus only on buyer's perspective. Crowther (1967), Monahan (1984) and Lal and Staelin (1984) were the first to consider the vendor's perspective by developing a model where the vendor offers discounts to the buyer to entice him to increase the batch size. Monahan's model considers a lot-for-lot policy (vendor does not carry any inventory) in which the vendor tries to maximize his resulting gains and compensates the buyer through discounting for the extra inventory holding charges that the buyer incurs. Lee and Rosenblatt (1986) extended the Monahan's model by relaxing the lot-for-lot assumption and hence allowing the vendor to carry inventory as well. Banerjee (1986) further extended the Monahan's model by incorporating a constant production rate for the vendor. Detailed reviews of such type of work can be found in Weng (1995) and Sarmah et al. (2006). Discounting models such as discussed by Li and Liu (2006) or by Shin and Benton (2007) have taken the final demand to be dependent on price and discuss a different philosophy of discounting which is different our focus.

The current paper examines the case of a dominant supplier whose optimal strategy is to set up only once for the production process in the planning horizon owing to his high set up cost. In such a case it is the buyer who has to carry the entire inventory. The buyer in turn offers the vendor a proposal of increasing the wholesale price as a form of compensation, so as to entice him to reduce the batch sizes and thus, can reduce his high own inventory holding costs. The case of dominant supplier with high cost of setting up of production where it is the vendor who decides on the lot sizes is quite common in many large industries like aerospace industry, automotive industry (Esmaeilli et al., 2009); as such 'reverse discount' mechanism of offering higher wholesale price by the buyer may be useful.

## 3. Mathematical Model

With the traditional assumption on the production cost structure of the vendor, it is understood that there is a fixed cost of 'set up' every time production is undertaken and there is a variable cost per unit produced. Further, with the assumption of infinite capacity of production, it is apparent that the optimal policy of the vendor facing a uniform deterministic demand will be to 'set up' only once in the entire planning horizon and also not to keep any inventory (note that the inventory is carried by the buyer rather than the vendor). For the buyer on the other hand the tradeoff between ordering costs and inventory costs may be such that it is often optimal to have more than one set up.

The proposed model assumes that the buyer can ask the vendor to increase the number of 'set ups' by offering him an increase in the wholesale price. Thus, he faces a tradeoff between an increase in the wholesale price and ordering costs on the one hand and a reduction in his inventory holding costs on the other hand. The vendor is being benefitted from the additional per unit wholesale price from the buyer but has to incur extra costs through the increased number of 'set ups'. The proposal from the buyer's side will only be accepted by the vendor if he is not worse off as compared to his original optimal plan. If the proposal from the buyer puts the vendor in a worse off position, he will reject such a proposal.

Consider a typical buyer-vendor scenario with the buyer facing a uniform deterministic demand. The costs for the vendor is the unit variable cost and set up cost per production run while for the buyer it is the unit variable cost of purchase, unit holding cost and ordering cost per order placed.

## 3.1. Notations:

w The wholesale price charged initially by the vendor (i.e. unit cost of purchase for the buyer)

c The unit variable cost incurred by the vendor to produce his goods

H The holding cost in \$ per unit \$ per unit time

$C_O$ The ordering cost of the buyer

$C_S$ Set up cost of the vendor

D Demand which is deterministic and uniform

As noted earlier, as the first step, the vendor decides, to meet the demand in one go. In the second step, the buyer comes up with a proposal of increase in wholesale price. The economics of the procedure is developed below:

*Step I*: When the vendor sets up only once and does not keep any inventory (Vendor's Optimal Decision)

Buyer's total cost= $wD + \frac{wHD}{2} + C_O = B_1$ (say) ……… (1)

Vendor's profit= $(w-c)D - C_S = S_1$ (say) ……… (2)

*Step II*: When the buyer wants to have a better bargain in terms of his total cost savings and comes up with the proposal of increasing the per unit variable price by an amount 'x' from the initial wholesale price with the underlying condition of asking the vendor for 'n' number of set ups for production in the entire planning horizon and in this scenario let 'Q' be the new EOQ

Then, Buyer's cost= $(w+x)D + (w+x)\frac{HQ}{2} + \frac{D}{Q}C_O = B_2$ (say) ……… (3)

Vendor's profit= $(w+x-c)D - \frac{D}{Q}C_S = S_2$ (say) ……… (4)

The proposal will be accepted to the vendor only if $S_2 \geq S_1$.

Moreover, for attaining feasibility we must have $B_2 \leq B_1$.

Let $\Delta_1 = B_1 - B_2 = wD + \frac{wHD}{2} + C_O - wD - xD - \frac{wHQ}{2} - \frac{xHQ}{2} - \frac{DC_O}{Q}$ ………. (5)

Substituting $D = nQ$ in the above expression we get

$$\Delta_1 = (n-1)\left(\frac{wHQ}{2} - C_O\right) - xQ\left(n + \frac{H}{2}\right) \geq 0 \qquad \text{………. (6)}$$

Also $\Delta_2$= $S_2$-$S_1$= $wD + xD - cD - \frac{D}{Q}C_S - wD + cD + C_S$

………. (7)

Substituting $D = nQ$ in the above expression we get

$$\Delta_2 = xnQ - (n-1)C_S \geq 0 \qquad \text{………. (8)}$$

The allowable range for x is given as $\left[\frac{(n-1)C_S}{nQ}, \frac{(n-1)\left(\frac{wHQ}{2} - C\right)}{Q\left(n + \frac{H}{2}\right)}\right]$

……….. (9)

whenever, $\frac{\left(\frac{wHQ}{2} - C\right)}{\left(n + \frac{H}{2}\right)} \geq \frac{C_S}{n}$ ………. (10)

This gives the 'feasibility set' (Binmore et al., 1986) for the bargaining process at which both the players (the buyer and the vendor as in our case) will agree.

The buyer will definitely select the lower bound for 'x' which will be acceptable to the vendor so that he can also alter his earlier plan, and on his proposal, the vendor will have to increase 'n' that will maximize his resulting profits. For a given increment in the wholesale price, the vendor's optimal decision will be to choose n=2.

When 'n' increases from 1 to 2, the resulting range of 'x' becomes $\left[\frac{C_S}{2Q}, \frac{\frac{wHQ}{2} - C_O}{Q\left(2 + \frac{H}{2}\right)}\right]$

$\left[\frac{\frac{wHD}{4} - C_O}{\left(2 + \frac{H}{2}\right)}\right] \geq \frac{C_S}{2}$ provided this is true. ……….. (11)

When the buyer chooses $x = C_S / D$, his net savings becomes $\frac{DwH}{4} - C_O - \frac{C_S}{2}\left(2 + \frac{H}{2}\right)$ ….. (12)

The optimal value for ‘x’ that will maximize the buyer’s cost savings subject to the condition that the vendor is not worse off can be found by solving the following optimization problem:

$$\left.\begin{aligned} &Max\Delta_1 = \frac{wHD}{2}\left(1 - \frac{1}{n}\right) - (n-1)C_O - \frac{xHD}{2n} - xD \\ &S.T.\ (n-1)C_S - xD \le 0 \end{aligned}\right\} \quad \text{……….. (13)}$$

### 3.2. Solution Procedure

In the above problem ‘n’ is an integer and hence techniques for solving non linear programming problems such as Karush-Kuhn Tucker conditions cannot be applied. Now suppose ‘n’ is a constant, then the problem

$$Max\Delta_1 = \frac{wHD}{2}\left(1 - \frac{1}{n}\right) - (n-1)C_O - \frac{xHD}{2n} - xD$$

$$S.T.\ x \ge \frac{(n-1)C_S}{D}$$

can be expressed as

$$Max\Delta_1 = \frac{wHD}{2}\left(1 - \frac{1}{n}\right) - (n-1)C_O - (n-1)C_S\left(1 + \frac{H}{2n}\right) \quad \text{....…… (14)}$$

since the expression $\frac{wHD}{2}\left(1 - \frac{1}{n}\right) - (n-1)C_O - xD\left(1 + \frac{H}{2n}\right)$ will attain its maximum value at the minimum value for ‘x’ given by $(n-1)C_S/D$.

Computing the value of the above expression for various values of ‘n’ we can find the point of maximum for ‘n’ and ‘x’ respectively (since the expression (14) is concave in ‘n’).

**Example:** Let us take w=25, H=0.05, D=5000, $C_O$=50 and $C_S$=500. The vendor, when decides to set up only once in the planning horizon, the buyer has to bear the entire inventory. The initial cost for the buyer= 251300. Keeping the value of 'n' fixed and computing the value of 'x' and evaluating the expression for maximum resulting gains for different values of 'n' we get

**Table 1**

Buyer's gain corresponding to different values of 'x' and 'n'

| **x** | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|
| **n** | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **Gains** | 15068.75 | 19725 | 21778.13 | 22790 | 23281.25 | 23475 | 23482.81 | 23366.67 | 23163.75 |

It can easily be seen from the above table, that the maximum gain is occurring for 'n'=8 giving the corresponding value of 'x' to be 0.07 and the resulting gain to be 23482.81. Fig. 1 shows the gain corresponding to different values of 'n' and other parameters. It should be also clear that each such pair (x, n); the supplier's profit remains unchanged.

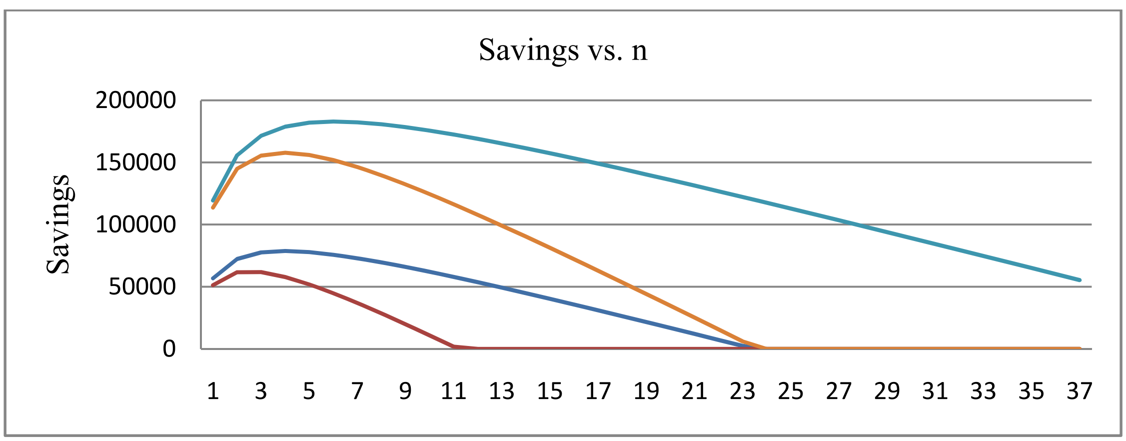


Fig. 1. Comparison of savings for different values of for different parameters

## 4. Case of Information Asymmetry

The lower and upper bound for increase in wholesale price that may be offered by the buyer has already been established in section 3, equation 9. Determination of the lower bound is only possible with the assumption that the buyer has the information related to the 'set up' cost of the vendor. In the absence of such information a proposal from the buyer may always have a chance of getting rejected by the vendor as it may not be cost effective for him. Under such a scenario the buyer will have to be contented with the initial 'threat point' solution of the bargaining process. But ideally, the buyer would like to offer the lower bound. Thus, absence of information may also lead to buyer offering an amount higher than the lower bound, leading to an opportunity loss for himself. It may be noted that there is no question of offering an amount higher than the upper bound as all information for determining the upper bound is known to the buyer. Additionally, the proposal of a price increase from the buyer might also be dependent on the risk taking ability of the buyer. A risk-averse buyer will try to offer a substantial price increase with the fear that a too low offer might be rejected by the vendor and in the process he will get reduced savings. A risk-taker buyer, on the other hand will try to offer a very low price increase and while he runs a risk of the proposal being rejected, he can have a significant gain if his estimation about the vendor's 'set up' cost is close to the original. A mathematical model is also presented below to illustrate as to how the risk taking capability of the buyer will influence his cost savings.

We assume that the buyer does not have the knowledge of the 'set up' cost of the vendor but knows that the 'set up' cost is uniformly distributed in the interval [a, b] where $a \leq C_S \leq b$. Such an assumption is in synchronization with Corbett et al. (2004). With this knowledge, the buyer calculates his expected additional gain and then decides on the price increase to be quoted that will maximize his additional gain. As already determined, the gain under initial condition is:

$$\Delta = \frac{wHD}{2}\left(1-\frac{1}{n}\right)-(n-1)C_O - xD\left(1+\frac{H}{2n}\right)$$

However there is a lower bound of x given by $x \geq \frac{(n-1)C_S}{D}$ and $C_S$ is uniformly distributed between [a, b].

Under information asymmetry related to the set up cost of the vendor, in case $C_S$ varies uniformly between [a, b], the decision problem for the buyer is thus

$$Max\Delta = (n-1)\left(\frac{wHD}{4}-C_O\right)-x\frac{D}{2}\left(n+\frac{H}{2}\right)$$

$$Subject\ to\ x \geq \frac{(n-1)C_S}{D}$$

*where* $C_S$ varies uniformly between [a, b] ……….. (15)

This is a non linear stochastic programming problem.

Let 'y' be the value of $C_S$ that will give the buyer maximum expected gain.

The expected gain is given by

$$\Delta = \left[ \frac{wHD}{2}\left(1-\frac{1}{n}\right) - (n-1)C_O - (n-1)\left(1+\frac{H}{2n}\right)y \right]\frac{y-a}{b-a} \qquad \text{……….. (16)}$$

Hence the problem is to find 'y' such that the value of the above expression is maximized.

The first order condition is:

$$\frac{\partial \Delta}{\partial y} = 0$$

$$\Rightarrow y = a + 1 + \left(\frac{wHD}{2n} - C_O\right) \Big/ \left(1+\frac{H}{2n}\right)$$

$$\therefore x = \frac{(n-1)}{D}\left[a + 1 + \left(\frac{wHD}{2n} - C_O\right) \Big/ \left(1+\frac{H}{2n}\right)\right]$$

Hence the quoted value of price increase will be

$$x = \frac{(n-1)}{D}\left[a + 1 + \left(\frac{wHD}{2n} - C_O\right) \Big/ \left(1+\frac{H}{2n}\right)\right] \qquad \text{………… (17)}$$

Again following the same solution procedure as is discussed in section 3, the maximum expected gains under the case of information asymmetry can be found.

## 5. Numerical Results

The various possible combinations of the different parameters in the model 1 were taken from the following set of values: w (25, 100, 500, 1000), H (0.01, 0.05, 0.1, 0.5), D (1000, 10000, 50000, 100000), $C_O$ (50, 100, 500, 1000) and $C_S$ (500, 1000, 5000, 10000) thereby giving a total number of 1024 possible combinations. Calculating the cost savings for all the 1024 situations, it was found that such a proposal of price increase from the buyer's side can reduce his costs by 6.95% on an average and which can even go up to 18% under some combinations of parameters.

Apart from these results it was also observed that the savings decrease with the increase in the $C_O$ to H ratio, savings increase with the increase the annual demand and also with the increase in the initial wholesale price. The variation of savings with respect to $C_O$/H is shown in Table 2 and Fig. 2 respectively.

**Table 2** Comparison of $C_O$/H with average savings

| $C_O$/H | % Savings |
|---|---|
| 100 | 7.192% |
| 200 | 7.192% |
| 500 | 7.176% |
| 1000 | 7.155% |
| 2000 | 7.155% |
| 5000 | 7.130% |
| 10000 | 7.130% |
| 20000 | 7.130% |
| 50000 | 7.130% |
| 100000 | 7.130% |

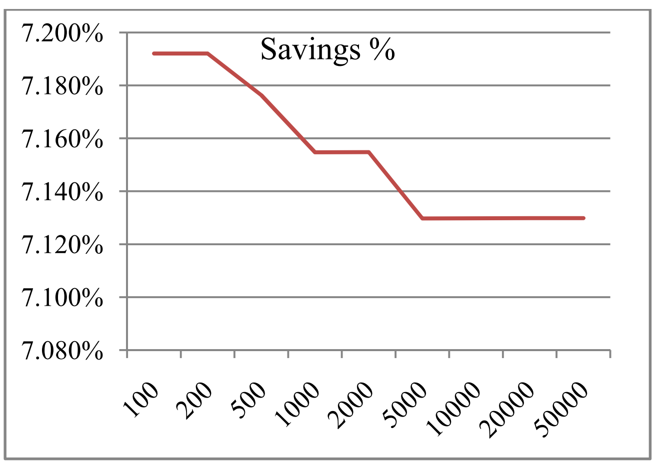


Fig. 2. Savings with respect to $C_O$/H

## 6. Conclusion

The above models show that not only a price discount but also a proposal of unit price increase from the buyer's side can be beneficial to both the partners involved in the transaction. The knowledge of full information related to costs of the vendor from the buyer's side will yield positive outcomes for both the partners. However, under information asymmetry, there is a trade off of between rejection of the proposal from the vendor's side and an opportunity loss from the buyer's side.

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## APPENDIX

Alternative Solution Procedure by using KKT conditions

Assuming ‘n’ to be continuous, KKT conditions can be applied to the expression in (13). We will first construct the Lagrangian function as:

$$L(x,n,\lambda) = -\frac{wHD}{2}\left[1-\frac{1}{n}\right] + C_O(n-1) + xD + \frac{xHD}{2n} + \lambda\left[(n-1)C_S - xD\right] \qquad \text{.......... (18)}$$

Derivation of the KKT conditions (Bazaraa et al., 1993)

$$\frac{\partial L}{\partial x} = D + \frac{HD}{2n} - \lambda D = 0 \Rightarrow \lambda = 1 + \frac{H}{2n}$$

$$\frac{\partial L}{\partial \lambda} = (n-1)C_S - xD = 0 \Rightarrow x = \frac{(n-1)C_S}{D}$$

$$\frac{\partial L}{\partial n} = -\frac{(w+x)HD}{2}\left(\frac{1}{n^2}\right) + C_O + \lambda C_S = 0 \Rightarrow \lambda = \frac{-C_O + \frac{HD}{2n^2}(w+x)}{C_S}$$

Solving the above optimization problem we get, $x = \frac{(n-1)C_S}{D}$ and

$$\lambda = \frac{H}{2n} + 1 = -C_O + \frac{HD}{2n^2}(w+x) \Big/ C_S$$

Since $-\frac{wHD}{2}\left[1-\frac{1}{n}\right] + C_O(n-1) + xD + \frac{xHD}{2n}$ is convex in nature, the KKT point will give the optimal solution.